arXiv:nucl-th/9807044v1 17 Jul 1998

Determination of ^{6}Li – ^{4}He interaction from multi-energy scattering data

R. S. Mackintosh and S.G. Cooper

Physics Department, The Open University,
Milton Keynes MK7 6AA

V.I.Kukulin

Institute of Nuclear Physics, Moscow State University, Moscow 119899,
Russia

r.mackintosh@open.ac.uk, s.g.cooper@open.ac.uk, kukulin@nucl-th.npi.msu.su

Abstract: We present the first successful potential model description of ^{6}Li – ^{4}He scattering. The differential cross-sections for three energies and the vector analyzing powers for two energies were fitted by a single potential with energy dependent imaginary components. An essential ingredient is a set of Majorana terms in each component. The potential was determined using a recently developed direct data-to-potential inversion method which is a generalisation of the IP S-matrix-to-potential inversion algorithm. We discuss the problems related to this phenomenological approach, and discuss the relationship of our results to existing and future theories.

PACS: 25.55.Ci, 25.55.-e, 24.10.Ht

February 9, 2008

Various studies of the scattering of unpolarised [1] and polarised [2, 3, 4] ^{6}Li from ^{4}He using conventional potential models have failed to yield a fully satisfactory description of the data. Conventionally parameterised potentials fail seriously, even when the absorption is allowed to be l-dependent. The inclusion of cluster transfer processes within the higher order DWBA framework [2] and the inclusion of coupling to projectile excitations [4], while clearly important, still do not result in satisfactory fits. In any case, the natural conclusion is still that the detailed description of nuclear scattering in terms of a potential model is not possible with ^{6}Li – ^{4}He scattering. In this paper we suggest that this is not the case.

Majorana terms are generally absent in published potential models of ^{6}Li – ^{4}He scattering. In a microscopic picture such as RGM these would arise, for example, from cluster exchange terms. In effect, this is what the DWBA or CRC cluster exchange calculations include in a less rigorous way. Green *at al* [2] show clearly how such processes enhance the backward angle scattering. It is well known that, within a potential model, Majorana terms lead to such enhancements. Determining the parameters of Majorana components is a non-trivial phenomenological problem since the radial form is unknown. In fact, we know from potentials which exactly reproduce RGM S-matrix elements for various systems that the radial forms of the Majorana terms will bear no simple relationship to those of the Wigner terms, see for example [5]. Clearly, one must seek both Wigner and Majorana terms which are unrestricted in form. However, model-independent fitting methods bring to light deep ambiguity problems which make it a necessary (but not always sufficient) part of the fitting procedure to constrain the potentials to be reasonably smooth and depend on energy in a physically reasonable way.

In the present paper we apply a recently developed technique, fulfilling the above requirements, to analyse ^{6}Li elastic scattering from ^{4}He. We have available numerical data for the following CM energies: 2.2 MeV and 7.85 MeV [3]; 11.1 MeV and 15.0 MeV [2]. For the 15 MeV case there is differential cross-section data only, but for all other cases vector and tensor analysing powers have been measured, although in this initial study we only fit the vector analysing powers. Moreover, for reasons to be given, the potentials we present here have not been fitted to the 2.2 MeV data.

The data is fitted by applying direct observable to potential inversion using a generalisation [6] of the iterative perturbative, IP, S-matrix to potential inversion method, a generalisation which has recently proven successful for the analysis of proton scattering from ^{16}O [7] and deuteron – ^{4}He scattering [8, 9]. The IP method for S-matrix to potential inversion has been described many times [10, 11, 12, 13], so we briefly outline the underlying concepts. The key idea is iteratively to correct a potential $V(r)$ by adding terms

$$V(r) \rightarrow V(r) + \sum c_i v_i(r) \tag{1}$$

where $v_i(r)$ are members of a suitable set of 'basis functions' and c_i are amplitudes derived from linear equations arising from the response, assumed linear, of the elastic scattering S-matrix to small changes δV in the potential:

$$\delta S_l = -\frac{\mathrm{i}m}{\hbar^2 k} \int_0^\infty (u_l(r))^2 \delta V(r) \mathrm{d}r. \tag{2}$$

In Equation 2, the radial wavefunction for angular momentum l is normalised according to $u_l(r) \rightarrow I_l(r) - S_l O_l(r)$ where I_l and O_l are the incoming and outgoing Coulomb

wavefunctions. The notation is simplified: $V(r)$ stands for real and imaginary, central and spin-orbit, Wigner and Majorana terms all of which can be expanded in different bases; for the treatment of spin and multiple energies see the papers cited above.

The generalised IP method enables direct observable to potential inversion [6, 7]. At each iteration, the potential amplitudes c_i in Equation 1 are determined by solving linear equations. These arise from the minimisation of the goodness of fit quantity χ^2,

$$\frac{\partial\chi^2}{\partial c_i} = 2\sum_{k,l}\left[\frac{\sigma_k - \sigma_k^{\rm in}}{(\Delta\sigma_k^{\rm in})^2}\right]\frac{\partial\sigma_k}{\partial S_l(E_k)}\frac{\partial S_l(E_k)}{\partial c_i} + 2\sum_{n,k,l}\left[\frac{P_{kn} - P_{kn}^{\rm in}}{(\Delta P_{kn}^{\rm in})^2}\right]\frac{\partial P_{kn}}{\partial S_l(E_k)}\frac{\partial S_l(E_k)}{\partial c_i}, \quad (3)$$

where $\sigma_k^{\rm in}$ and $P_{kn}^{\rm in}$ are the input experimental values of cross sections and analyzing powers respectively (n indexing the spin related observables for spin 1 systems), and

$$\chi^2 = \sum_{k=1}^{N}\left(\frac{\sigma_k - \sigma_k^{\rm in}}{\Delta\sigma_k^{\rm in}}\right)^2 + \sum_n\sum_{k=1}^{M}\left(\frac{P_{kn} - P_{kn}^{\rm in}}{\Delta P_{kn}^{\rm in}}\right)^2. \quad (4)$$

Since we are fitting data for several, possible many, energies at once, the index k indicates the energy as well as angle. For brevity, we shall refer below to this method for observable to potential inversion as the 'Generalised Iterative Perturbative', GIP, method. For recent applications to d + ^{4}He scattering, including the determination of phase shifts, refer to Refs [8, 9, 14].

An essential ingredient in the GIP approach is the 'starting reference potential', SRP, from which the iterative process starts. This is of great importance in a system which is plagued by ambiguities even when fits are precise. It is the main opportunity for the inclusion of *a priori* information derived from physical insight. This is essential for systems such as that under discussion, for which there is no possibility of true model independent fitting of the kind associated, for example, with electron scattering, although we see no reason not to aspire to fits of such quality.

We seek eight potential components in all: real and imaginary central and vector spin orbit terms, each a sum of Wigner and Majorana terms: $V_{\rm W}(r) + (-1)^l V_{\rm M}(r)$. For the real potential, the inversion determines the radial shape of the potential and the coefficients of a polynomial expansion in energy which multiplies the radial form. That is, the algorithm determines $V(r)$ and ξ_i in the expression:

$${\rm Re}V(r,E) = V(r)(1 + \xi_1 E + \xi_2 E^2 + \ldots). \quad (5)$$

(More general forms are allowed by the algorithm but are not relevant to this work.) The imaginary potential is also parameterised as the product of a radial form to be determined and a function of energy. The energy function takes the form of a leading power term plus a polynomial series which is not used in most cases described here. The form was determined by the requirement with certain cases of light nuclei that the imaginary part be zero below the reaction threshold, E_0. The leading term is:

$${\rm Im}V(r,E) = W(r)\left(\frac{E - E_0}{E_{\rm ref} - E_0}\right)^p \quad (6)$$

so that in the present case, taking $E_0 = 0$ and absorbing the other constants into the radial form, we have simply an E^p dependence. The inversion procedure does

not automatically adjust p. In the present case we have fixed $p = 1$ but there is no restriction in principle on p to integral or positive values.

All the potentials we present were found using a Gaussian inversion basis, generally starting with three or four terms for each component. There is a natural limit to the desirable basis dimension since, as this is increased, there comes a point where there is a rather sudden onset of oscillations in the potential. Only vector spin-orbit interactions were included and no attempt was made to fit tensor analysing powers. A coupled channel extension of IP inversion to determine tensor potentials is presently under development [15]. Studies with deuteron scattering suggest that, although omission of $T_{\rm R}$ terms might compromise the vector interaction somewhat, the general features of the central potential should be little affected.

Preliminary investigations. Our initial calculations involved only the two higher energies. We first used as SRP various the potentials given by Green *et al* [2]. In this way we achieved our best overall fit to the 11.1 MeV and 15 MeV data. However, we discount this potential as a possible physical solution since the volume integral per nucleon pair for the real central Wigner component, $J_{\rm R}$, was about 800 MeV fm^3, much higher than the volume integral that might be expected from systematics. Indeed, ^{6}Li scattering phenomenology suggests a potential which is weaker in the surface, as a result of breakup processes, than the M3Y folding model [16] potential, see below. It seems likely that Green *et al* [2] found potentials with very large $J_{\rm R}$ as a consequence of attempting to fit aspects of the data which cannot be fitted without Majorana components. Of the various terms of the SRP, it is the choice of the real central Wigner term, by far the largest component, which most influences the final potential.

Subsequent investigations. For the fits that we present here the central Wigner component of the SRP was the M3Y density independent folding model [16] potential incorporating a ^{6}Li density of Suelze *et al* [17]. The imaginary and spin-orbit components were taken from Green *et al*. We first found a potential which simultaneously fitted the $E_{\rm CM} = 11.1$ and 15.0 MeV data, and then found a simultaneous fit to the $E_{\rm CM} = 7.85$, 11.1 and 15 MeV data. In the following figures and text, these data are referred to by the projectile (^{6}Li) energies, 19.6, 27.7 and 37.5 MeV, respectively. We comment below on the 5.5 MeV ($E_{\rm CM} = 2.2$ MeV) data.

First, we fitted the 27.7 and 37.5 MeV data from FSU with a potential for which the real Wigner central component had a volume integral $J_{\rm R} = 339$MeV fm^3, i.e. somewhat below that for the M3Y folding model, but still much closer to it than the initial potentials with $J_{\rm R} = 800$ MeV fm^3 [2]. This potential, which we refer to as 2EN-1, is included for comparison in Figures 3 – 5 to be referred to below. It had no pathologies such as unitarity breaking in particular lj partial waves. This potential did not give a satisfactory fit to the Wisconsin [3] data.

An attempt to fit the two Wisconsin data sets together was not very successful, and it seemed that the very low energy, 5.5 MeV, data were too easily fitted. The backward angle oscillations in the other cases are evidence of two amplitudes interfering, and this requires Majorana terms in a single particle model. However, there are other ways of getting the structure-less backward angle rise at 5.5 MeV, and this seems to have undermined the fitting process.

In Figures 1, 2 and 3, we present a simultaneous fit to the 19.6, 27.7 and 37.5 MeV data. The most serious mismatch is to the forward angle 37.5 MeV data although this is less serious than for the 'two-energy' fit. A suggestion of such a forward angle mismatch is present in an existing fit [2] to the 37.5 MeV data, and it was decided not to pursue perfect fits at these angles. The imaginary terms were energy dependent (except the Wigner SO term) as described above with $p = 1$, $E_0 = 0$ and $E_{\rm ref} = 19.6$. The real terms (not spin-orbit) turned out to be very weakly energy dependent, ξ_1 being very small. The potential corresponding to Figures 1 to 3 is referred to as 3EN-1 in Figures 4 and 5 which also show the 'two-energy potential' labelled 2EN-1, and an alternative potential, labelled 3EN-2. The real, central, Wigner component of the adopted potential 3EN-1 has volume integral $J_{\rm R} = 411$ MeV fm^3 and rms radius 3.042 fm.

The potential 3EN-2 is more oscillatory than the others, and the fit is markedly poorer, especially for 19.6 MeV. It was found in an attempt to eliminate unitarity breaking by 3EN-1 for $l = 2$ and $j = 3$ at 19.6 MeV. Although 2EN-1 did not show such a pathology, it is, on balance, a less reasonable representation of the data. With discrete two-step data fitting, i.e., data $\rightarrow S_{lj}$ followed by $S_{lj} \rightarrow V(r)$ inversion, it should be possible to constrain $|S| \leq 1$; the next stage in the development of the present method will be to incorporate such a constraint.

Unitarity breaking is thus a symptom of the difficulties presented by these data, presumably due to strong non-localities and channel couplings. The ambiguity problems were greater than presented by d + ^{4}He data, although these were significant [8, 9]. The question of what we can say reliably about the potential is not straightforward. Should we completely discard a potential which breaks unitarity in one lj channel? In the present case $S_{23} = 1.26$, unquestionably an unphysical feature. However, we argue that the key properties of the potential are determined. The problem is that, because one must carefully limit the dimension of the inversion basis, it is unlikely that a potential which does precisely fit the data will lie within the space spanned by the particular basis employed. It appears that potentials which are found to give reasonable fits may, among their many components, have an emissive region which happens to coincide with a substantial value of $|\psi|^2$ for some particular lj leading to $|S| > 1$ for that lj. Many inversion studies with RGM S-matrix elements have shown that emissive regions certainly do occur in local potentials representing the RGM S-matrix (with all $|S| \leq 1$, however) as a result of non-locality arising from exchange and channel coupling effects.

Whether or not this picture can be substantiated, we have shown that there exists a potential, having reasonable central Wigner real and imaginary terms and also substantial Majorana terms, which does give *much* better fits than other potential models. The fact that there also exists a potential which gives a better fit to just the 27.7 and 37.5 MeV data but which is certainly unphysical (having perhaps twice the expected $J_{\rm R}$) shows two things: (i) the ambiguity problem is deep and treacherous, and (ii) the OM fits in the original papers found unreasonably deep potentials because they were trying to fit the backward angles without Majorana terms. This last point implies that, if we have shown nothing else, we have shown that ^{6}Li + ^{4}He scattering is compatible with the expectations of folding models (as long as some account of exchange via Majorana terms or otherwise is included.) To emphasise the reasonableness of the potential, we compare in Figure 6 the real central Wigner terms for the 3EN-1 and 2EN-1 potentials

with the M3Y density independent folding model potential incorporating a ^{6}Li density of Suelze *et al* [17]. It is clear that the three energy potential 3EN-1 is remarkably close to the folding model potential except in the surface; just where one expects repulsive effects due to the breakup of ^{6}Li, see Ref. [10] and papers cited therein. The alternative folding model[1] potential shown in Figure 8 of Kamal *et al* [18] is just a little deeper than 3EN-1 but remarkably similar in shape to both it and the M3Y potential.

One conclusion of this work is that it is *essential* in this sort of fitting to include *a priori* knowledge, in this case by employing an M3Y based folding potential as starting potential (SRP). Two complementary sources of such information are: (i) RGM that includes exchange exactly and, by way of inversion of RGM S_{lj}, that yields useful starting forms for central and vector spin-orbit Wigner and Majorana terms; (ii) $\alpha + d + \alpha$ cluster models with d-exchange, which can include the D-state components which are omitted in current RGM calculations. These latter will be useful at the next step of this research programme, i.e. when tensor interactions are included and tensor analysing powers are fitted. How might our analysis be affected by our omission of tensor degrees of freedom? We explored this with test cases involving deuterons with S_{lj} calculated with potentials which included a strong $T_{\rm R}$ interaction. Inversion of these S_{lj} gave potentials with just central and vector spin-orbit interactions and also Majorana components. Though not a rigorous argument, this suggests that the central Wigner terms are little affected by the omission of tensor interactions from the analysis, but that the spin-orbit interactions and Majorana terms will be modified to some extent. This corresponds to the fact that differential cross-sections are hardly modified by tensor interactions, which do, however, somewhat affect vector analysing powers.

We can now draw the following conclusions: (i) it is possible to get a reasonable simultaneous fit to the scalar and vector observables for three energies with a single energy dependent potential; (ii) the real, central, Wigner term is very close to what would be expected from a folding model; (iii) substantial Majorana terms must be included, even though their specific nature might be obscured by the neglect of tensor forces; (iv) one cannot assume the Majorana terms to be of the form $1+\alpha(-1)^l$ times the real components. In cases where RGM or cluster exchange theories imply the existence of Majorana terms, they must be included in phenomenological analyses. If they are omitted, one cannot achieve the quality of fit which is necessary for determining even the Wigner components. Thus, the best fit pure Wigner potentials, which did not fit the data very well, had much larger $J_{\rm R}$ values than folding models would imply. A significant limitation at present, and a reason for not pressing for closer fits, is the assumption that the energy dependent potentials are of fixed radial form.

The GIP method has much scope for development as a powerful tool for analysing nuclear scattering. Elsewhere [8], we have described its utility for phase shift analysis in the common situation where the data is incomplete. The next developments will be the introduction of unitarity constraints and tensor interactions. If successful, the quantitative evaluation of microscopic theories will become possible. This is of great interest for $\alpha+^6$Li for which there are reasons to anticipate parity dependent tensor forces. Such theories are very hard to test by conventional methods. This is because, on the

[1]Based not on an effective NN-force like M3Y but on well established cluster-cluster potentials, correctly fitted to the data.

one hand, the ambiguity and fitting problems faced by conventional phenomenology are extreme, and, on the other hand, because the complexity of the theory for this ten body system is such that even the most elaborate *ab initio* scattering calculations are likely to give modest fits to the data, making direct evaluation of the theory problematic. This is certainly the case for the d + ^{4}He six nucleon[2] scattering system [6, 9, 14, 19], and will be even more so here. However, when the present method is applied with *a priori* information in the form of SRPs derived by inversion from theory, and with the inversion potential constrained to depart as little as possible from the SRP, then the final departure from the SRP required to get a perfect fit will furnish a quantitative evaluation of the theory.

When it was first found long ago that, contrary to expectations, nucleon scattering could be described by a single particle model, it was considered remarkable. We have shown that the range of application of this model is by no means exhausted, but a necessary generalisation is that all components must be free to be parity dependent.

Acknowledgements

We are much indebted to K. Kemper of FSU and E. George of Wisconsin for supplying us their impressive numerical data. We are also grateful to the Engineering and Physical Science Research Council of the UK for grants supporting S.G. Cooper and to the Royal Society for supporting a visit by Professor Kukulin.

References

[1] H.G. Bingham, K.W. Kemper and N.R. Fletcher, Nucl. Phys. **A 175** (1971) 374

[2] P.V. Green, K.W. Kemper, P.L. Kerr, K. Mohajeri, E.G. Myers, D. Robson, K. Rusek and I.J. Thompson, Phys. Rev. **C53** (1996) 2862

[3] E.A. George, D.D. Pun Casavant and L.D. Knutson, Phys. Rev. **C56** (1997) 270

[4] K. Rusek, P.V. Green, P.L. Kerr and K.W. Kemper, Phys. Rev. **C56** (1997) 1895

[5] R. S. Mackintosh and S.G. Cooper, Nucl. Phys. **A589** (1995) 377; S.G. Cooper and R.S. Mackintosh, Nucl. Phys. **A592** (1995) 338

[6] V.I. Kukulin, V.N. Pomerantsev and S.B. Zuev, Yad. Fiz. **59** (1996) 428; English translation: Physics of Atomic Nuclei, **59** (1996) 403

[7] S.G. Cooper, Nucl. Phys. **A 618** (1997) 87

[8] S.G.Cooper, V.I.Kukulin, R.Mackintosh, and E.V. Kuznetsova, Phys. Rev. C. in press; nucl-th/9803014

[2]One problem is including the deuteron and alpha particle D-states within the fully antisymmetrized RGM formalism. It is the deuteron D-state, of course, which leads to the $T_{\rm R}$ interaction in the inter-nucleus potential.

[9] R.S. Mackintosh and S.G. Cooper, contribution to International Workshop on Physics with Radioactive Nuclear Beams, Puri, January 1998, nucl-th/9803016; to appear in J. Phys. G.

[10] R.S. Mackintosh and A.M. Kobos, Phys. Lett. **116B** (1982) 95; A.A. Ioannides and R.S. Mackintosh, Nucl. Phys. **A438** (1982) 354.

[11] V.I. Kukulin, V.N. Pomerantsev and J. Horaček, Phys. Rev. **A 42** (1990) 2719

[12] S.G. Cooper and R.S. Mackintosh, Phys. Rev. **C 43** (1991) 1001; Nucl. Phys. **A517** (1990) 285; Phys. Rev. **C 54** (1996) 3133

[13] S.G. Cooper and R.S. Mackintosh, Inverse Problems, **5** (1989) 707

[14] V.I. Kukulin, V.N. Pomerantsev, S.G. Cooper and S.B. Dubovichenko, Phys. Rev. C**57** (1998) 2422

[15] S.G. Cooper, V.I. Kukulin, V.N. Pomerantsev and R.S. Mackintosh, in preparation.

[16] G.R. Satchler and W.G. Love, Phys. Reports **5** (1979) 183

[17] L.R. Suelze, M.R. Yearian and H. Crannell, Phys. Rev. **162** (1967) 992

[18] M.Kamal, V.T.Voronchev, V.I.Kukulin, V.M.Krasnopolsky, Y.Nakao and K.Kudo, J.Phys. G: **18** (1992) 379.

[19] R.S. Mackintosh and S.G. Cooper, Nucl. Phys. **A 625** (1997) 651

Figure Captions

Figure 1. Fit to ^{6}Li – ^{4}He differential cross-section and analysing power data, for incident ^{6}Li energy 19.6 MeV, with parity dependent and energy dependent potential 3EN-1.

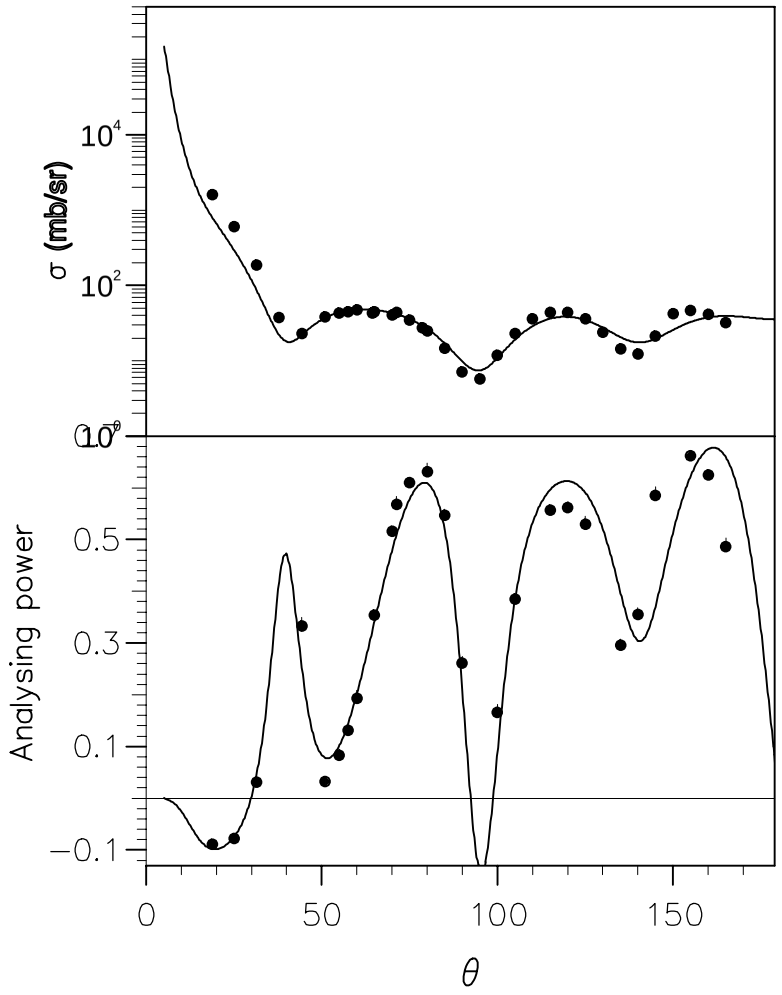
Figure 2. Fit to ^{6}Li – ^{4}He differential cross-section and analysing power data, for incident ^{6}Li energy 27.7 MeV, with parity dependent and energy dependent potential 3EN-1.

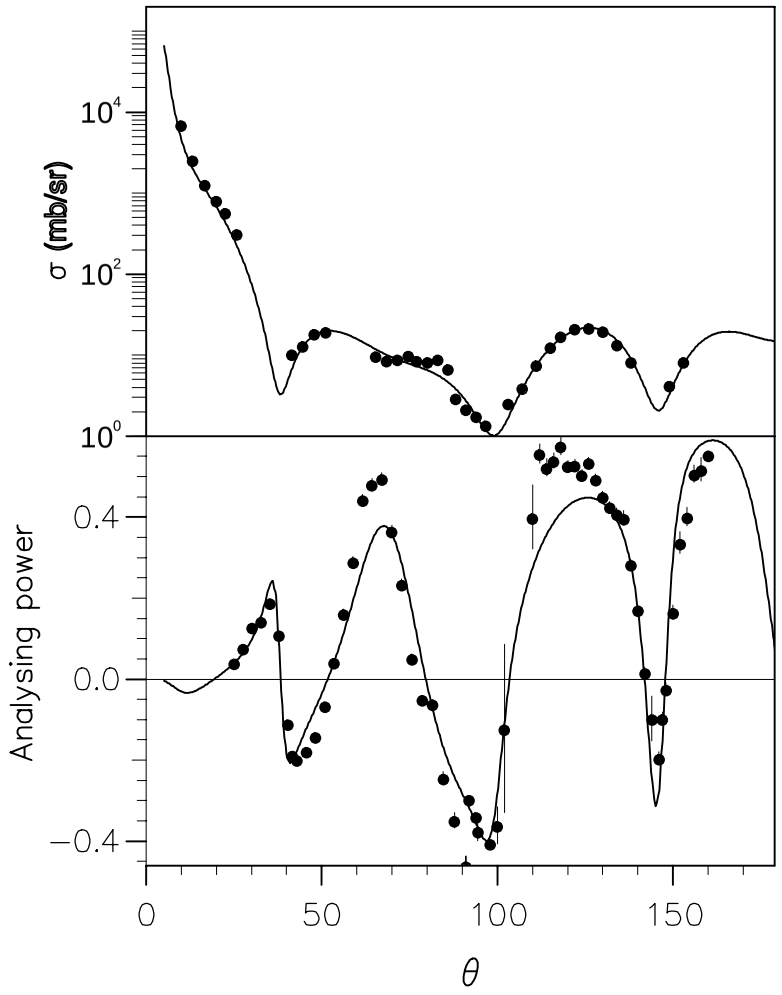
Figure 3. Fit to ^{6}Li – ^{4}He differential cross-section data, for incident ^{6}Li energy 37.5 MeV, with parity dependent and energy dependent potential 3EN-1.

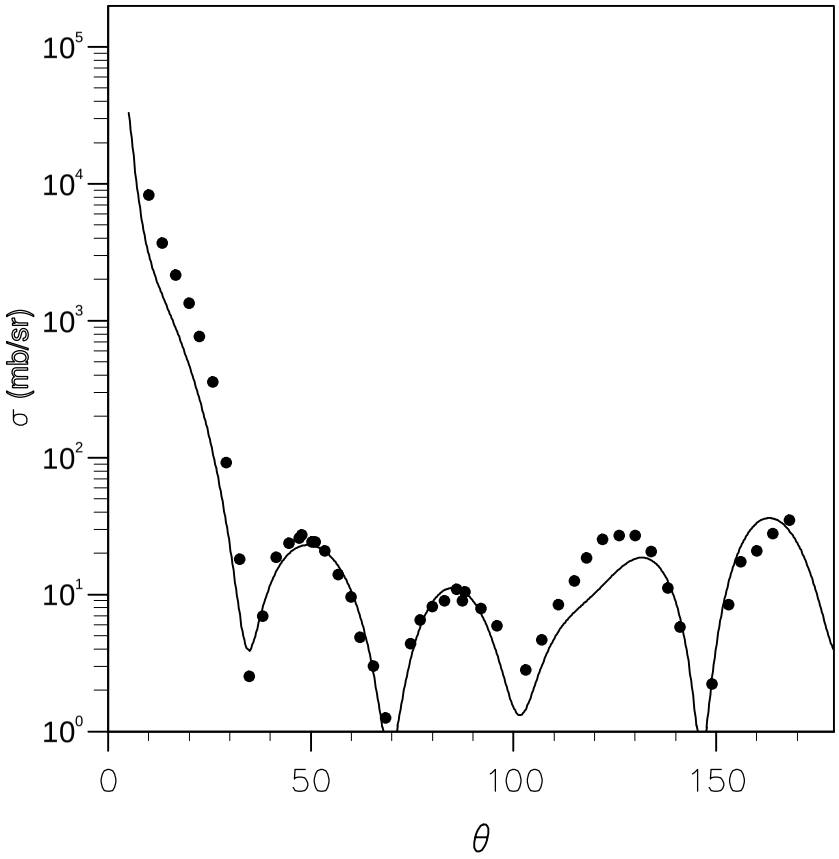
Figure 4. Wigner components of potentials fitting ^{6}Li – ^{4}He scattering data. From the top, real and imaginary central, real and imaginary spin-orbit. The solid line is 2EN-1 fitting data for the two higher energies; the dashed line is the best 'three energy' potential, 3EN-1, and the dotted line represents 3EN-2, an unsuccessful attempt to improve the fit, showing the tendency to become oscillatory. The imaginary terms are evaluated for $E = E_{\rm ref}$.

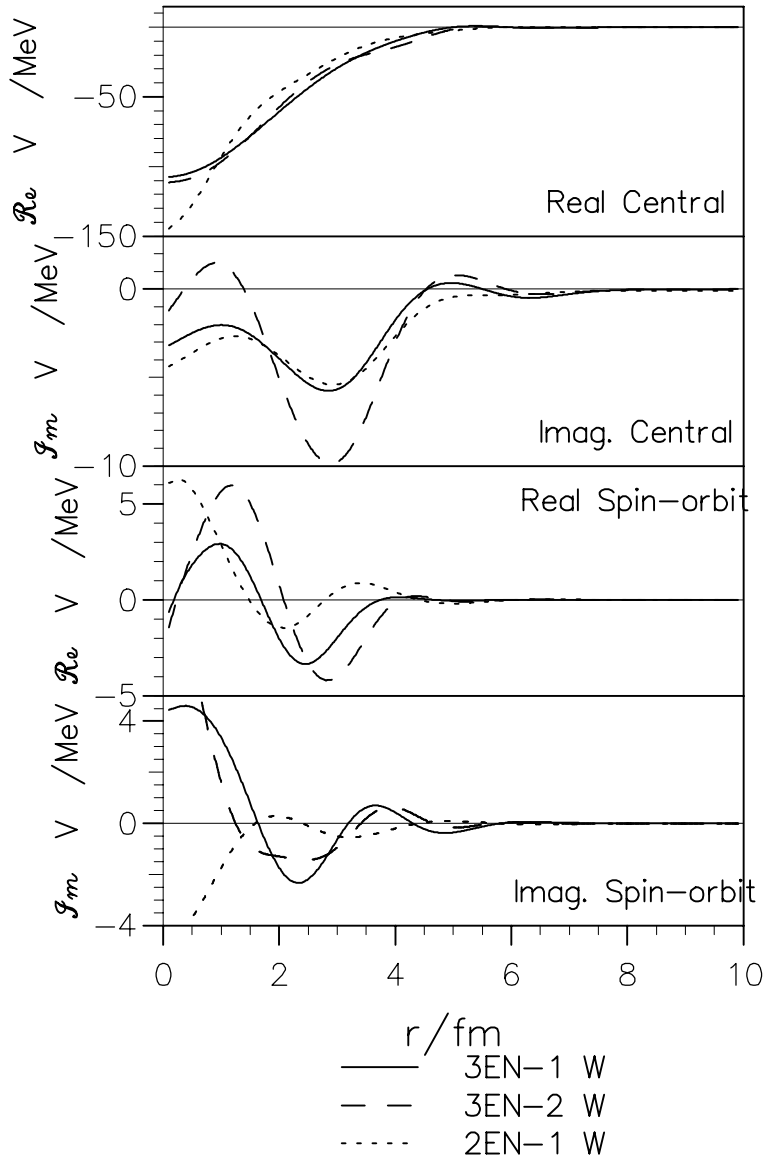
Figure 5. Majorana components as for Figure 4.

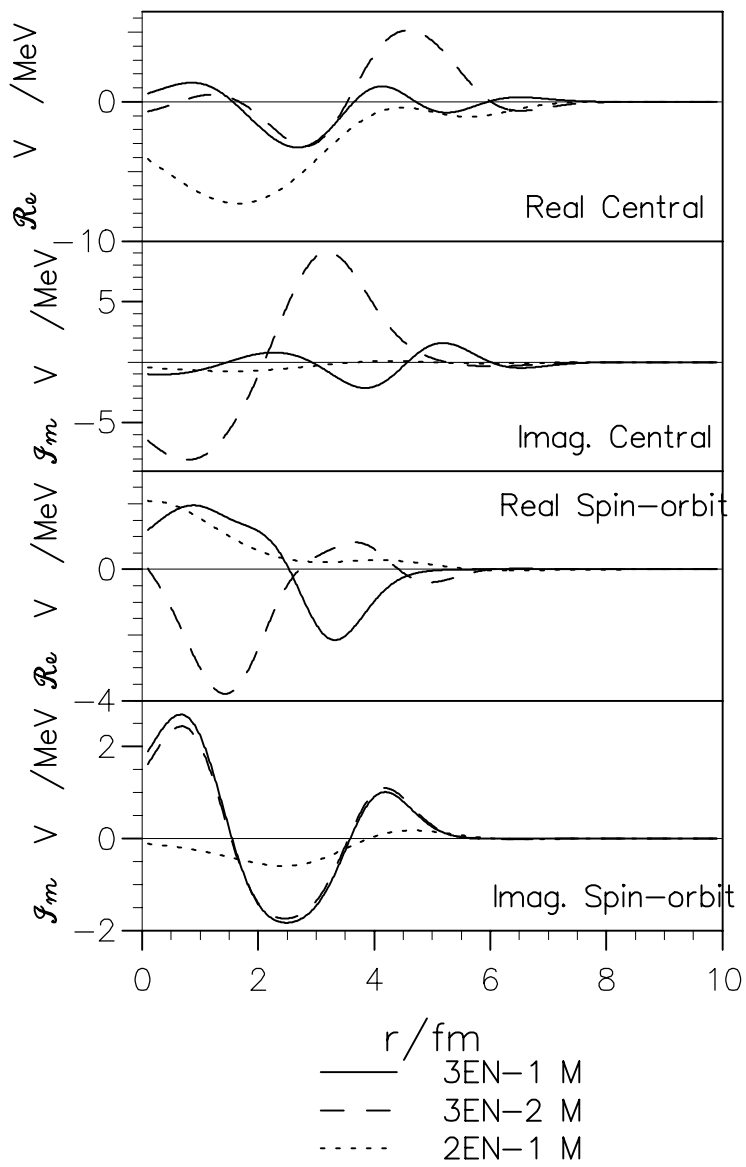
Figure 6. Comparing the real, central, Wigner component of the potentials 3EN-1 and 2EN-1 with the M3Y folding model potential.

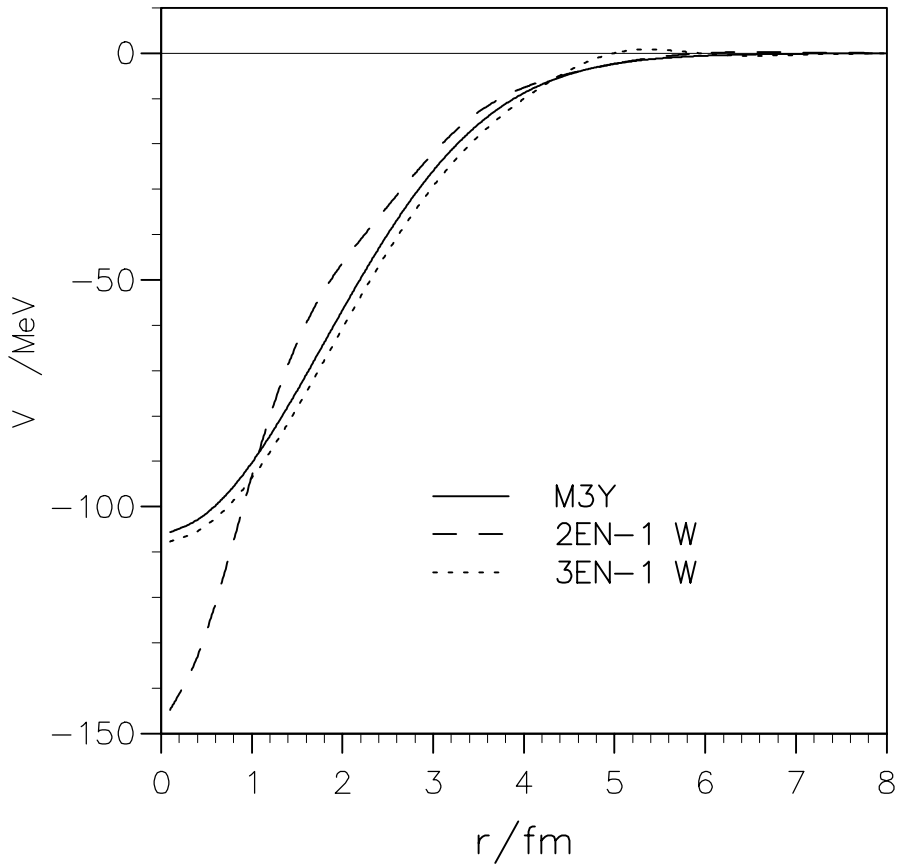

σ (mb/sr)
10^4
10^2
10^0
Andlysing power
0.5
0.3
0.1
−0.1
0
50
100
150
θ


σ (mb/sr)
10^4
10^2
10^0
Andlysing power
0.4
0.0
−0.4
0
50
100
150
θ


10^5
10^4
10^3
10^2
10^1
10^0
σ (mb/sr)
0
50
100
150
θ


Re V /MeV
−50
−150
Real Central
Im V /MeV
0
−10
Imag. Central
Re V /MeV
5
0
−5
Real Spin−orbit
Im V /MeV
4
0
−4
Imag. Spin−orbit
0
2
4
6
8
10
r/fm
3EN−1 W
3EN−2 W
2EN−1 W


Re V /MeV
0
−10
Real Central
Im V /MeV
5
−5
Imag. Central
Re V /MeV
Real Spin−orbit
0
−4
Im V /MeV
2
0
−2
Imag. Spin−orbit
0
2
4
6
8
10
r/fm
3EN−1 M
3EN−2 M
2EN−1 M


0
−50
−100
−150
V /MeV
0
1
2
3
4
5
6
7
8
r/fm
M3Y
2EN−1 W
3EN−1 W